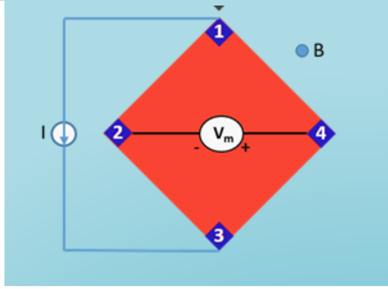
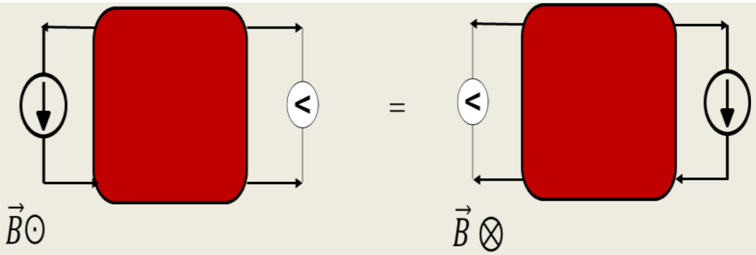
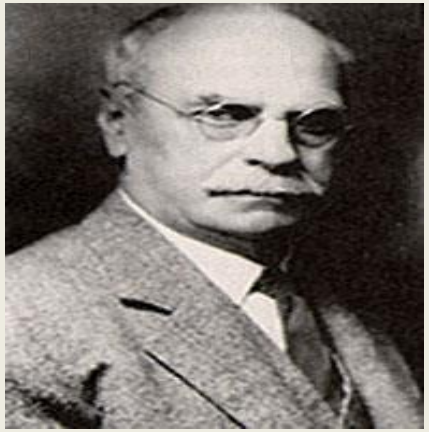
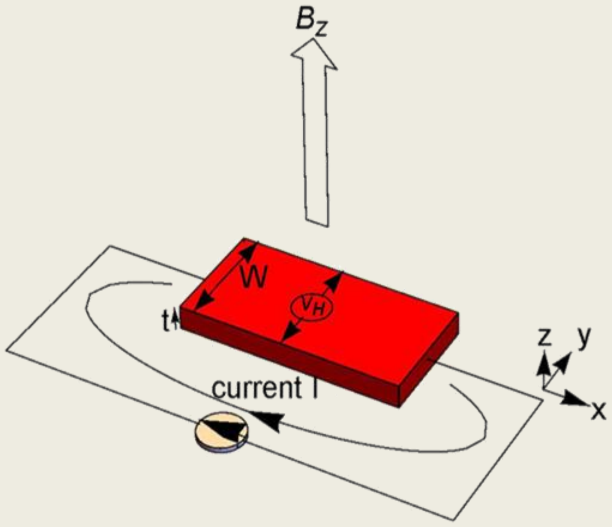
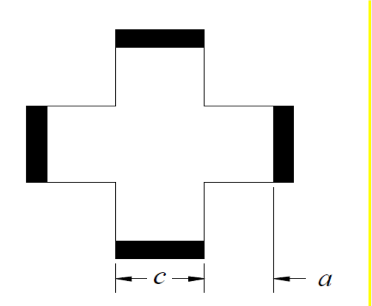






Edwin Hall 1855 – 1938





A Faster Hall Measurement

Jeffrey Lindemuth

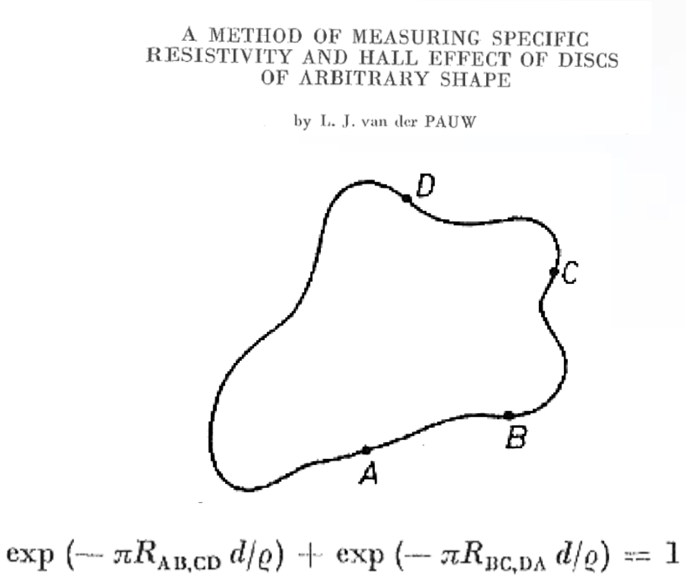
Applications Scientist

Outline

- Fundamentals of Hall effect measurements
  - DC Hall effect
  - AC Hall effect
- A novel approach to Hall effect characterization

Hall effect characterization

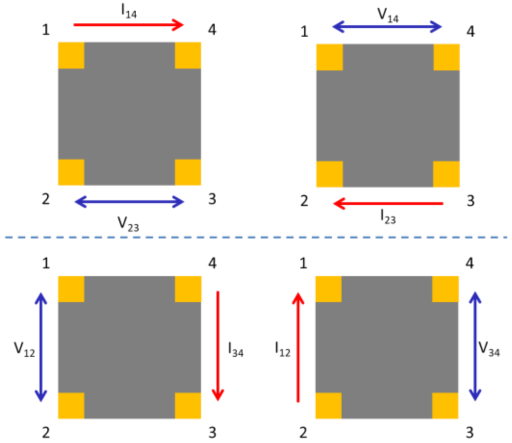
Resistivity: ρ



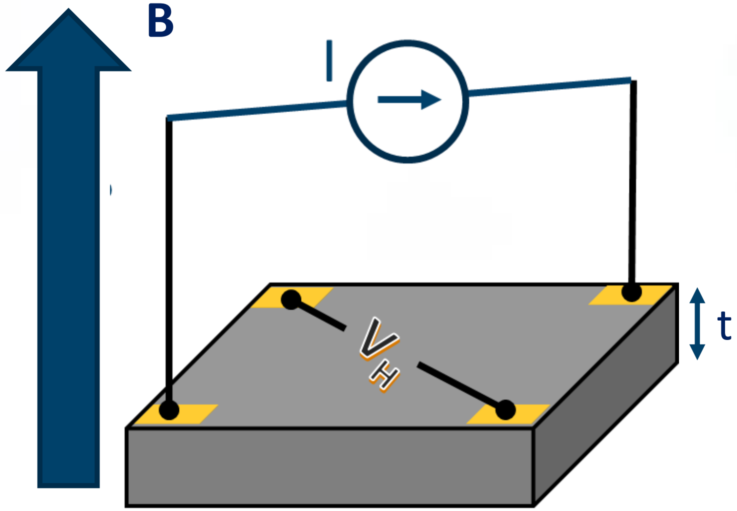
A METHOD OF MEASURING SPECIFIC
RESISTIVITY AND HALL EFFECT OF DISCS
OF ARBITRARY SHAPE

by L. J. van der PAUW

$$\exp(-\pi R_{\mathrm{AB,CD}}\, d/\varrho) + \exp(-\pi R_{\mathrm{BC,DA}}\, d/\varrho) = 1$$



Hall effect



$$V_H = \frac{IB}{nqt} = \frac{IR_H B}{t}$$

- Carrier type from sign of Hall voltage
- Carrier density from:

$$n = \frac{IB}{V_H q t}$$

- Hall mobility from:

$$\mu_H = \frac{|R_H|}{\rho}$$

Hall effect measurements

- There are 4 key factors in a Hall measurement:
  - **Hall voltage (V_H)**
    - The value we want to measure
    - Calculate Hall coefficient $R_H=V_Ht/IB$ and mobility $\mu=R_H/\rho$
  - **Misalignment voltage (V_M)**
    - Error due to imperfect sample geometry
    - Not field dependent
    - Use field reversal to remove by subtraction
  - **Thermal electric voltage (V_{TE})**
    - Error due to temperature gradients
    - Not current dependent
    - Use current reversal to remove by subtraction
  - **Noise and offset**
    - Error due to experimental & instrumentation limitations

Hall voltage

Misalignment voltage

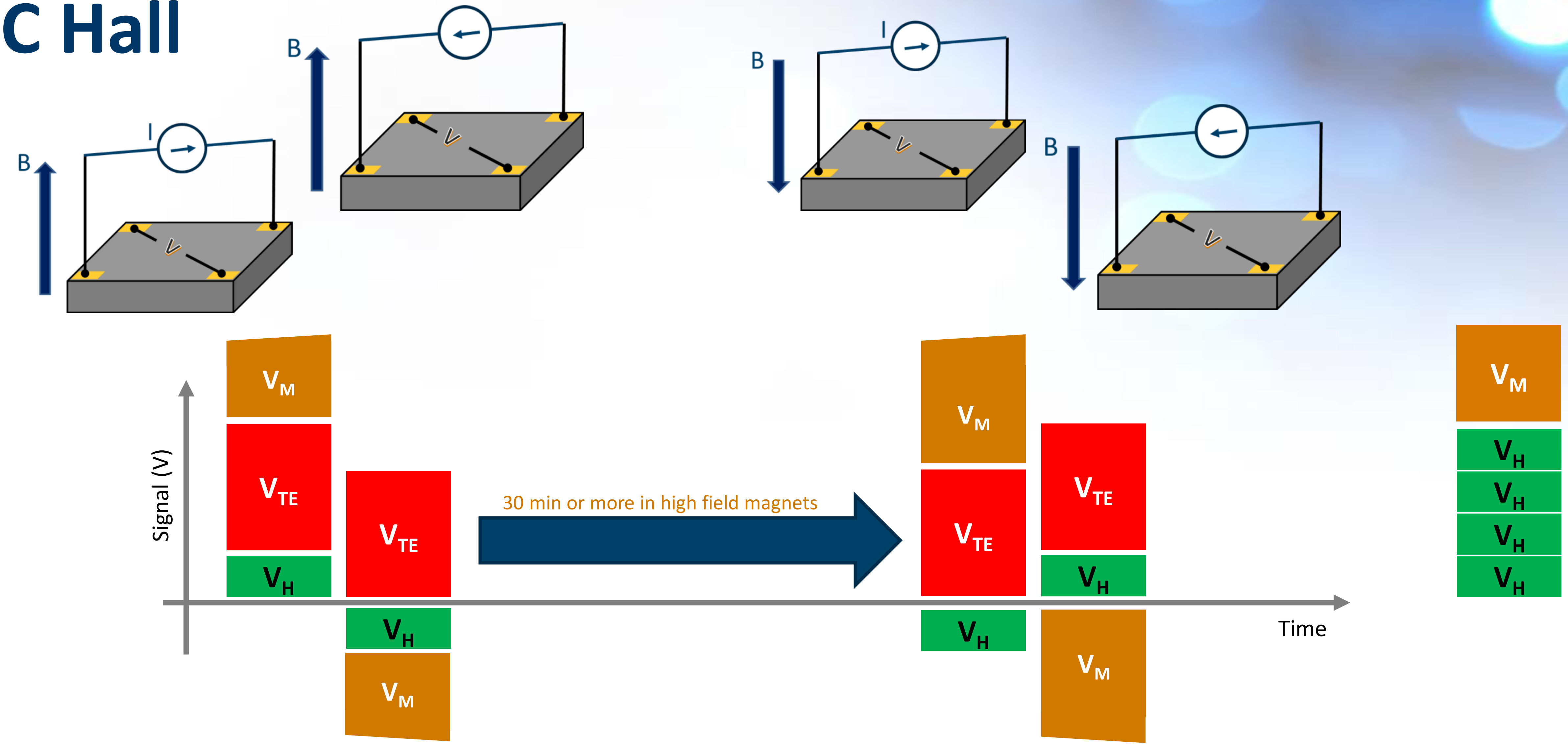
T/E voltage

Noise

- 3 minor factors in a Hall measurement:

- Nernst effect voltage (V_N)
- Righi-Leduc effect voltage (V_R)
- Ettingshausen effect voltage (V_E)

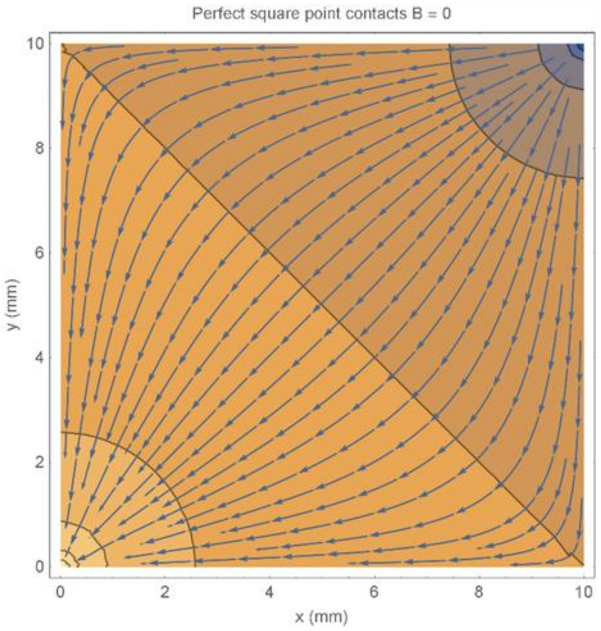
DC Hall



When the misalignment voltage is large compared to the Hall voltage, cancellation of the misalignment voltage with field reversal will not be complete due to voltage fluctuations and/or drift.

What is the misalignment voltage and how to control it?

The misalignment voltage, the measured voltage in a hall measurement at zero field, is often the largest intrinsic error in a Hall measurement. It is a purely geometric effect. If the sample was a perfect square and the contacts were mathematical point contacts on the corner of the sample the misalignment voltage would be zero. Any deviation from this ideal will result in non-zero misalignment voltage.

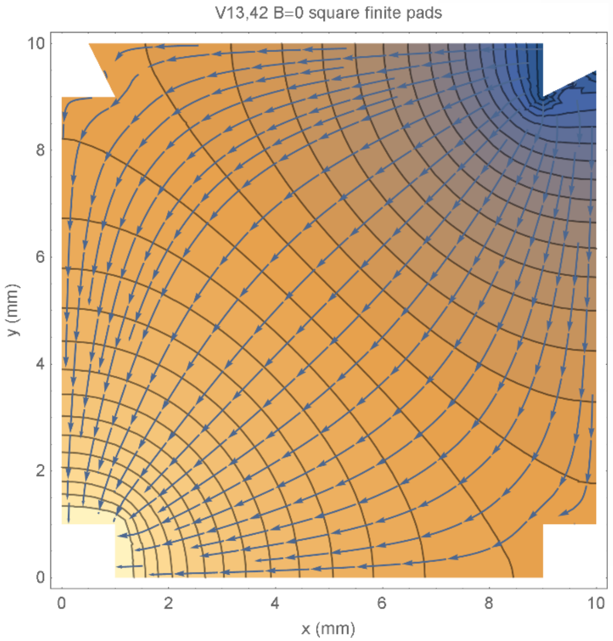


This figure shows the equipotential lines and current flow for a perfect square sample with point contacts.

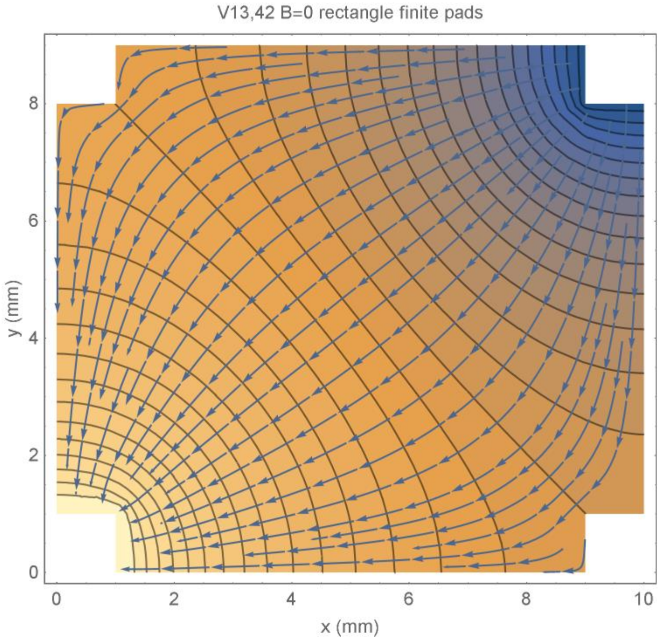
The misalignment voltage in this case is 0.

The modeled sample has a sheet resistivity of 1000 ohm/sqr and a mobility of 1 $cm^2/(V\ s)$.

The measurement current is 1 mA, and the Hall voltage for this sample at B = 1 T is 100 µV.



The same sample but with finite deformed contact size. In this case the misalignment voltage is 0.066 V for the same excitation.



The same sample but the length to width ratio is 0.9, with finite contact size. In this case the misalignment voltage is 0.127 V.

AC field Hall

When the B field becomes an AC field,

DC voltage

$$V_{measured} = \underbrace{R_H I B cos(\omega t)}_{V_H} + \overset{\downarrow}{V_M} + \overset{\downarrow}{V_{TE}} - \underbrace{\beta B \omega \sin(\omega t)}_{\text{Inductive term } \propto \frac{dB}{dt}}$$

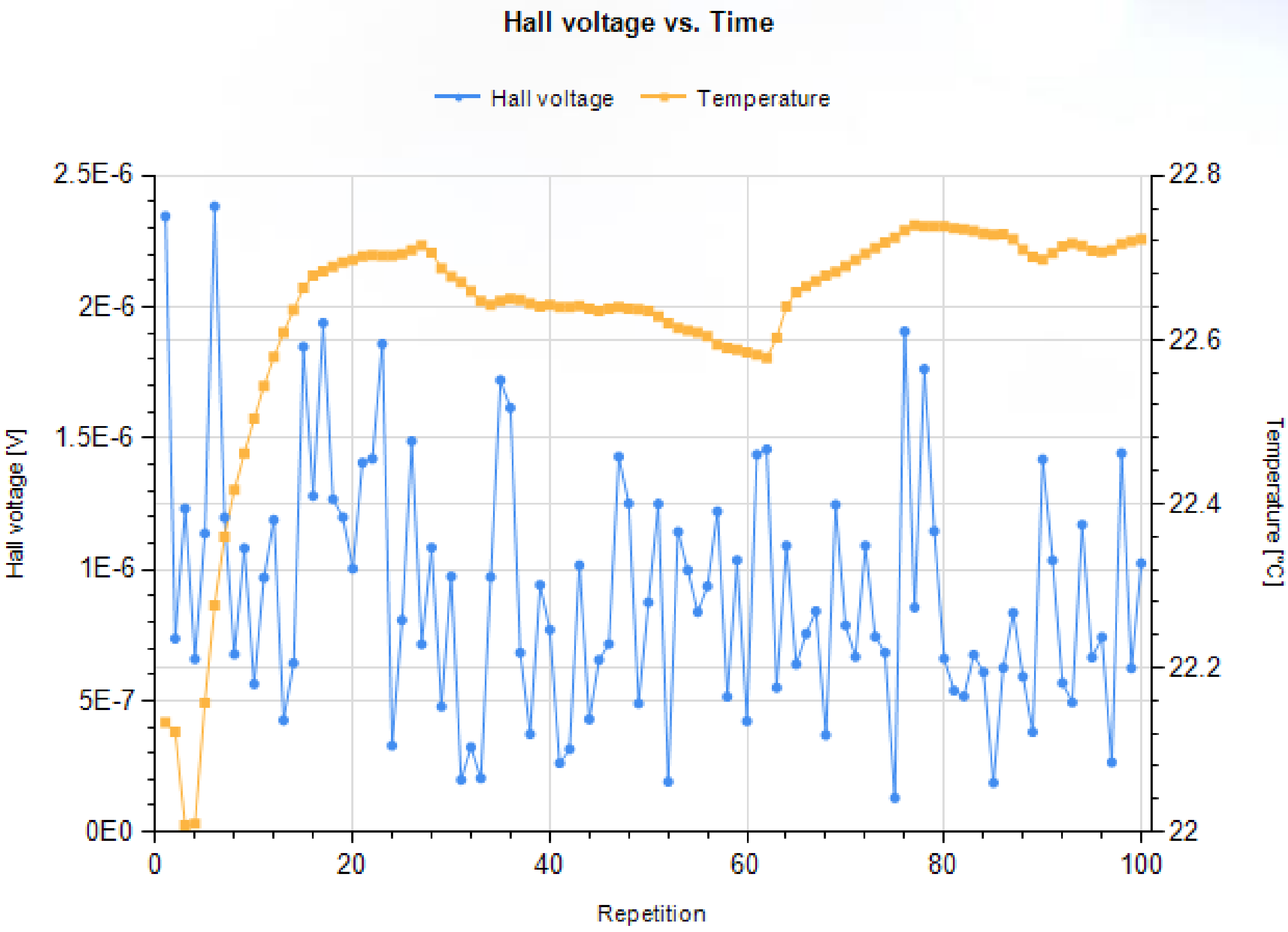
V_H

- Remove DC components with lock-in amplifier

Inductive term $\propto \frac{dB}{dt}$

- At quadrature
- ω must be small (0.1 Hz)
- Slow measurements

AC Hall of organic semiconductor P3HT



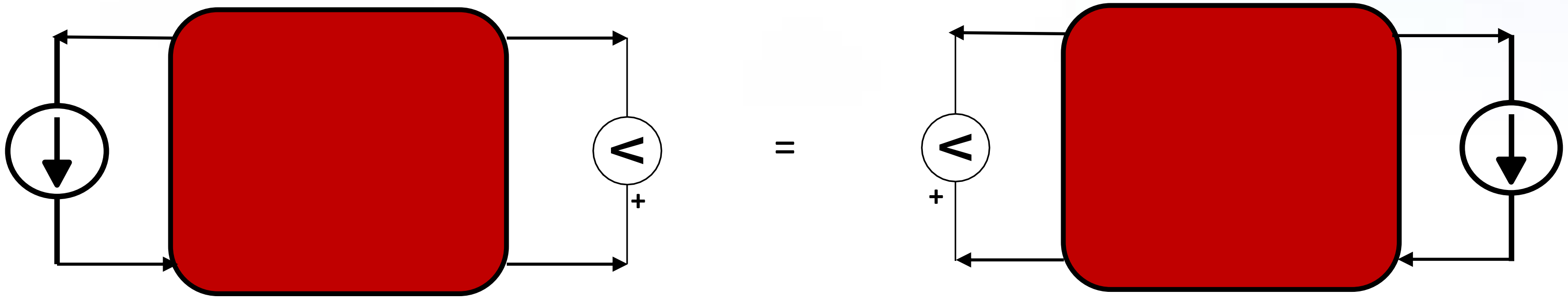
- < 1 µV Hall signal
- 100 samples
- approximately 10-hour measurement
- ~0.7 °C temperature drift over the measurement

Thanks to Benjamin Schwartz of UCLA for providing the P3HT sample.

Is there a better way?

Reciprocity

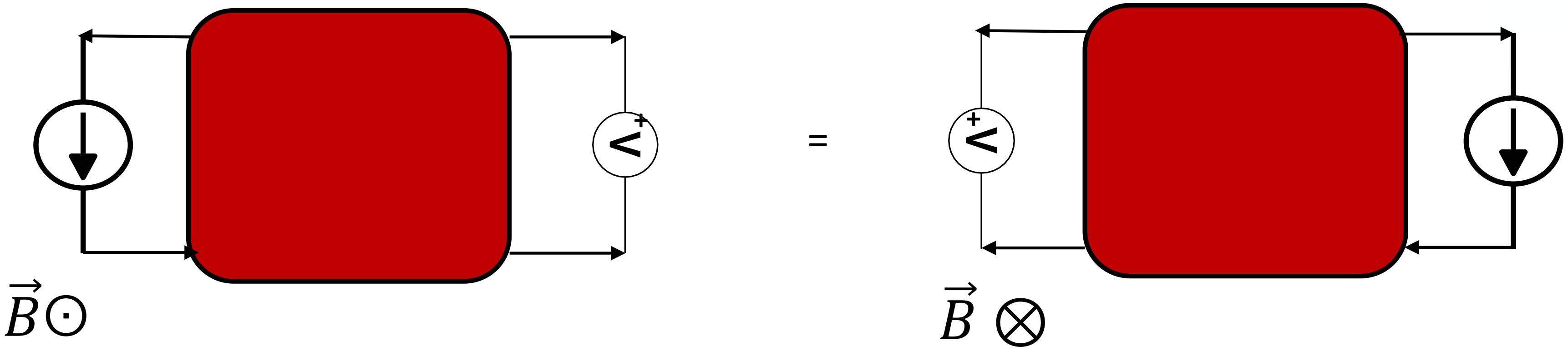
- Well known reciprocity theorem from electrical network theory
- Currents and voltages at different points in the network can be interchanged:

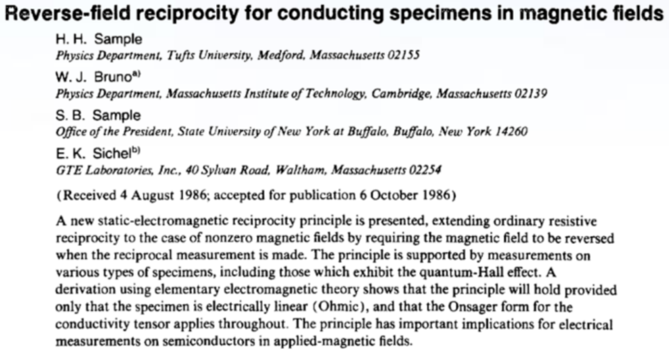


In the above example, the voltage reading, for the same current input is the same in both diagrams
This only requires that the network (sample) be linear and passive.

Reverse-field reciprocity

- The above theorem applies for no external magnetic field. If there is an external field ($\vec{B}$) then the reciprocity theorem is that the voltage and current can be interchanged and the direction of the magnetic field is reversed





Reverse-field reciprocity for conducting specimens in magnetic fields

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Physics Department, Tufts University, Medford, Massachusetts 02155
W. J. Bruno[a)]
Physics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
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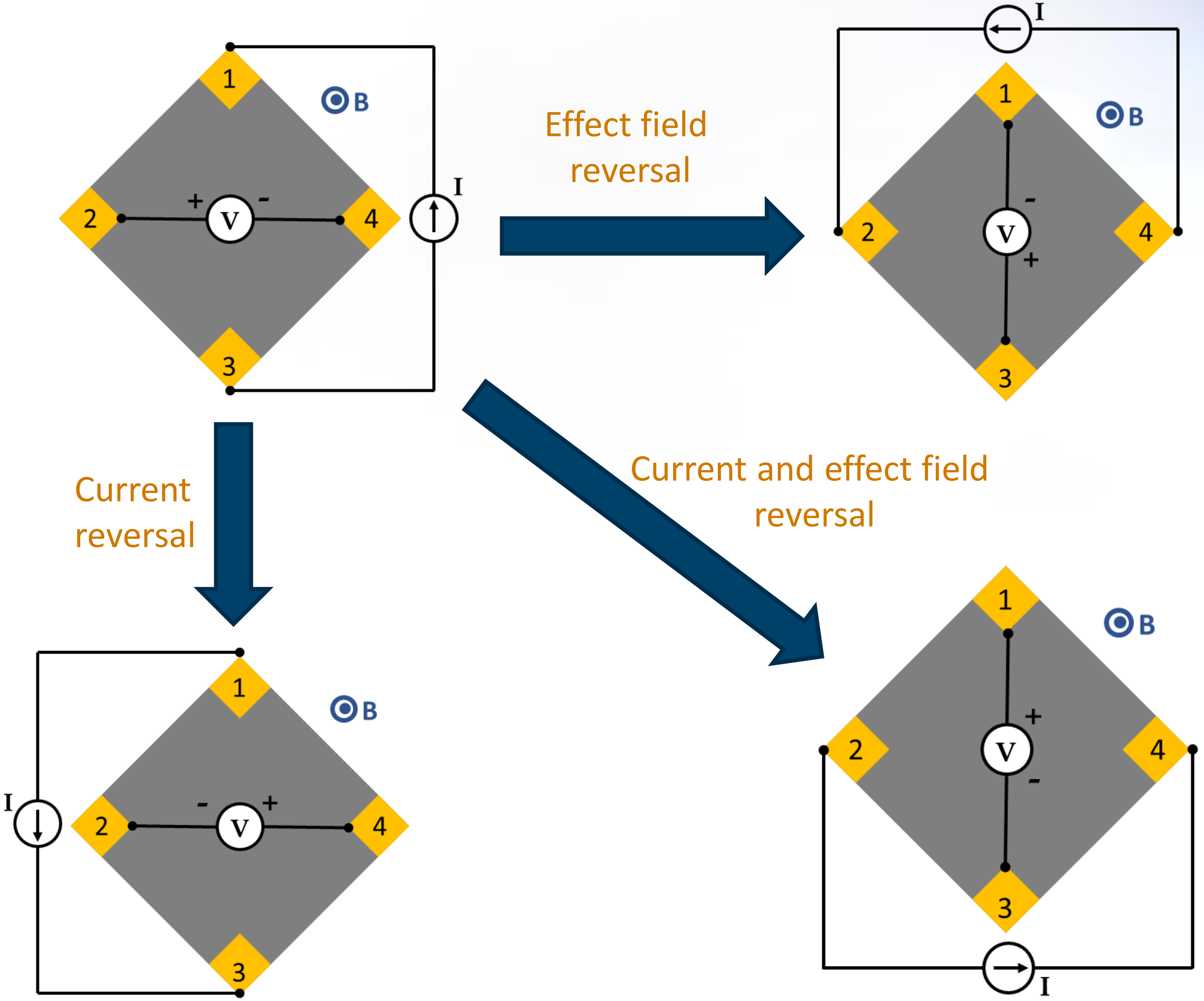
(Received 4 August 1986; accepted for publication 6 October 1986)

A new static-electromagnetic reciprocity principle is presented, extending ordinary resistive reciprocity to the case of nonzero magnetic fields by requiring the magnetic field to be reversed when the reciprocal measurement is made. The principle is supported by measurements on various types of specimens, including those which exhibit the quantum-Hall effect. A derivation using elementary electromagnetic theory shows that the principle will hold provided only that the specimen is electrically linear (Ohmic), and that the Onsager form for the conductivity tensor applies throughout. The principle has important implications for electrical measurements on semiconductors in applied-magnetic fields.

J. Appl. Phys. **61** (3), 1 February 1987

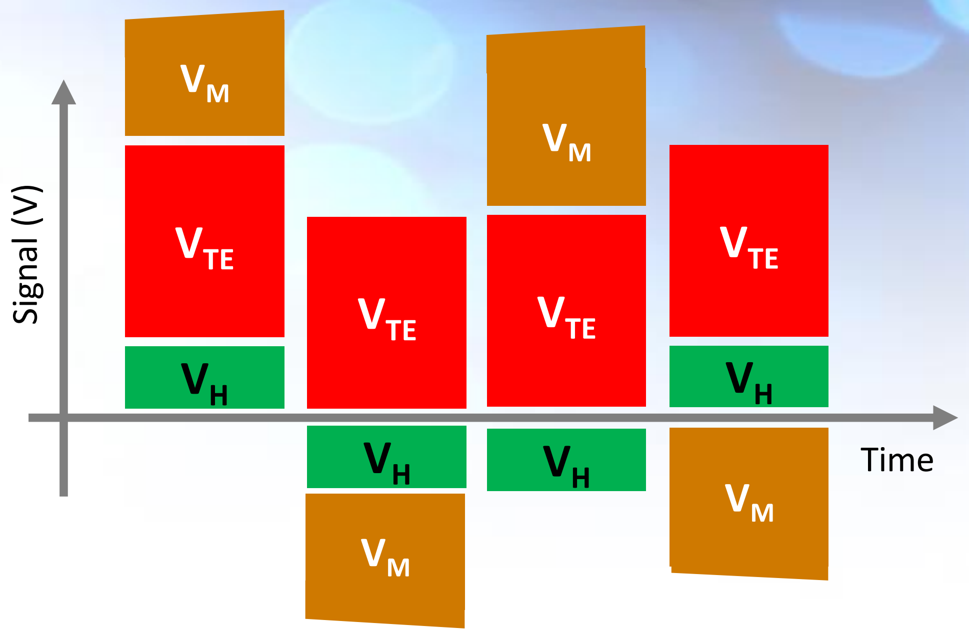
The only restriction on the network (sample) is that it is linear (in current and voltage) and passive. In particular, the sample does not have to be uniform or symmetric. It does require ohmic contacts with no thermoelectric voltages. Thermoelectric voltages can be handled, carefully, by current reversal.

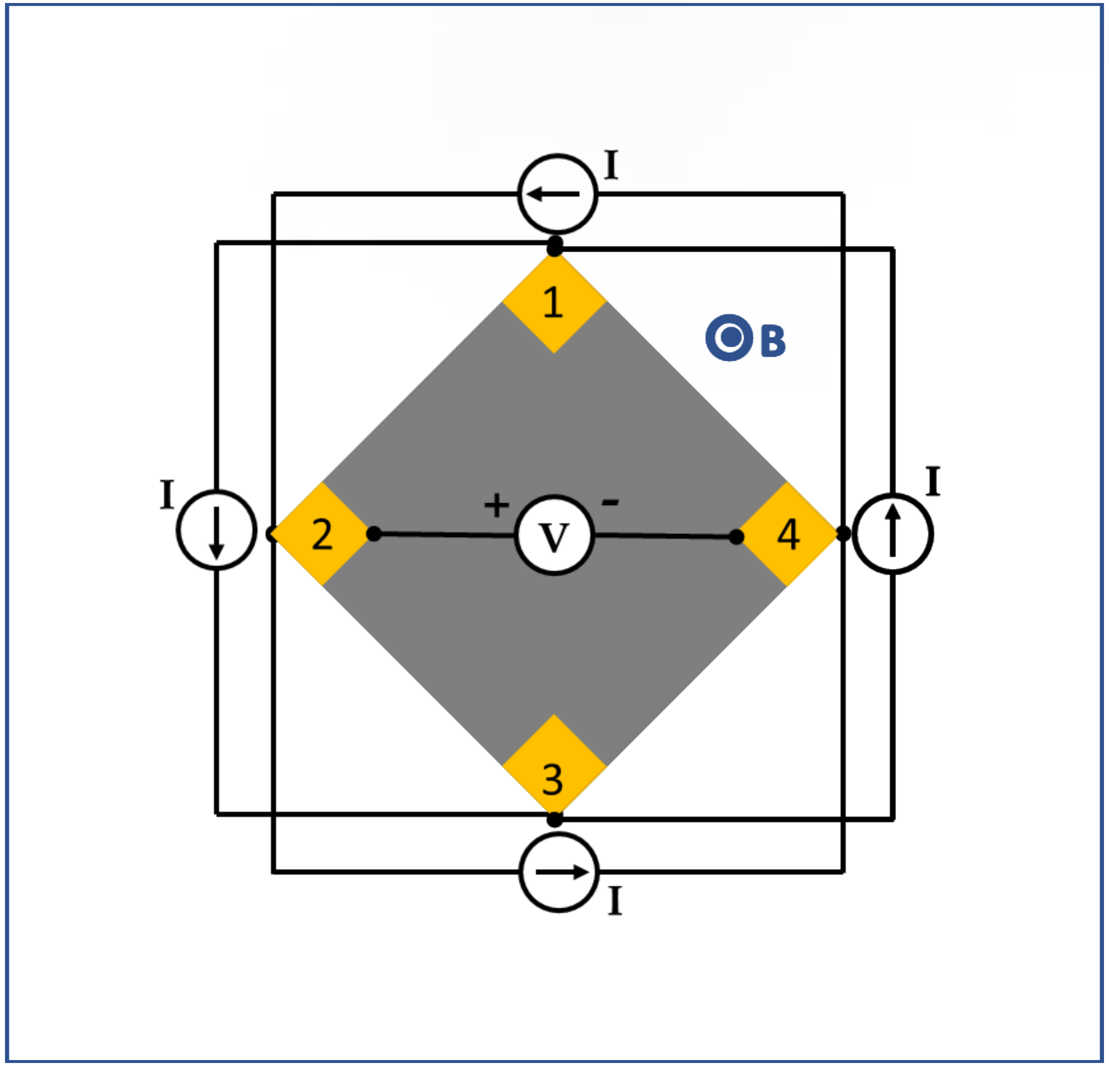


Effect field reversal

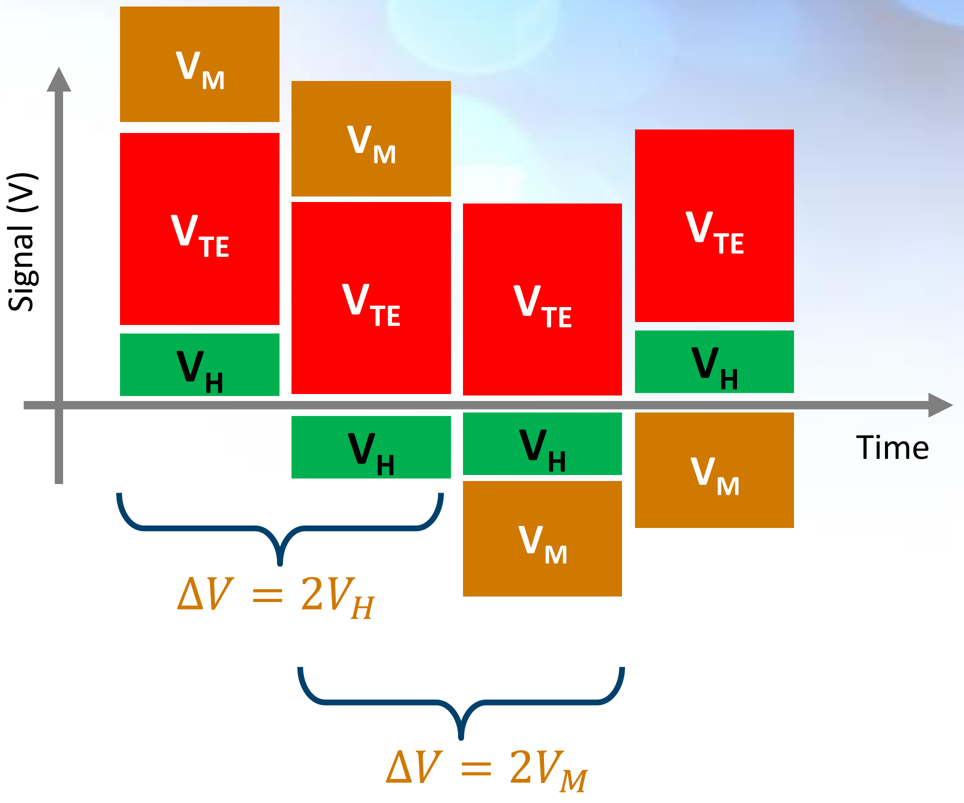
Current reversal

Current and effect field reversal



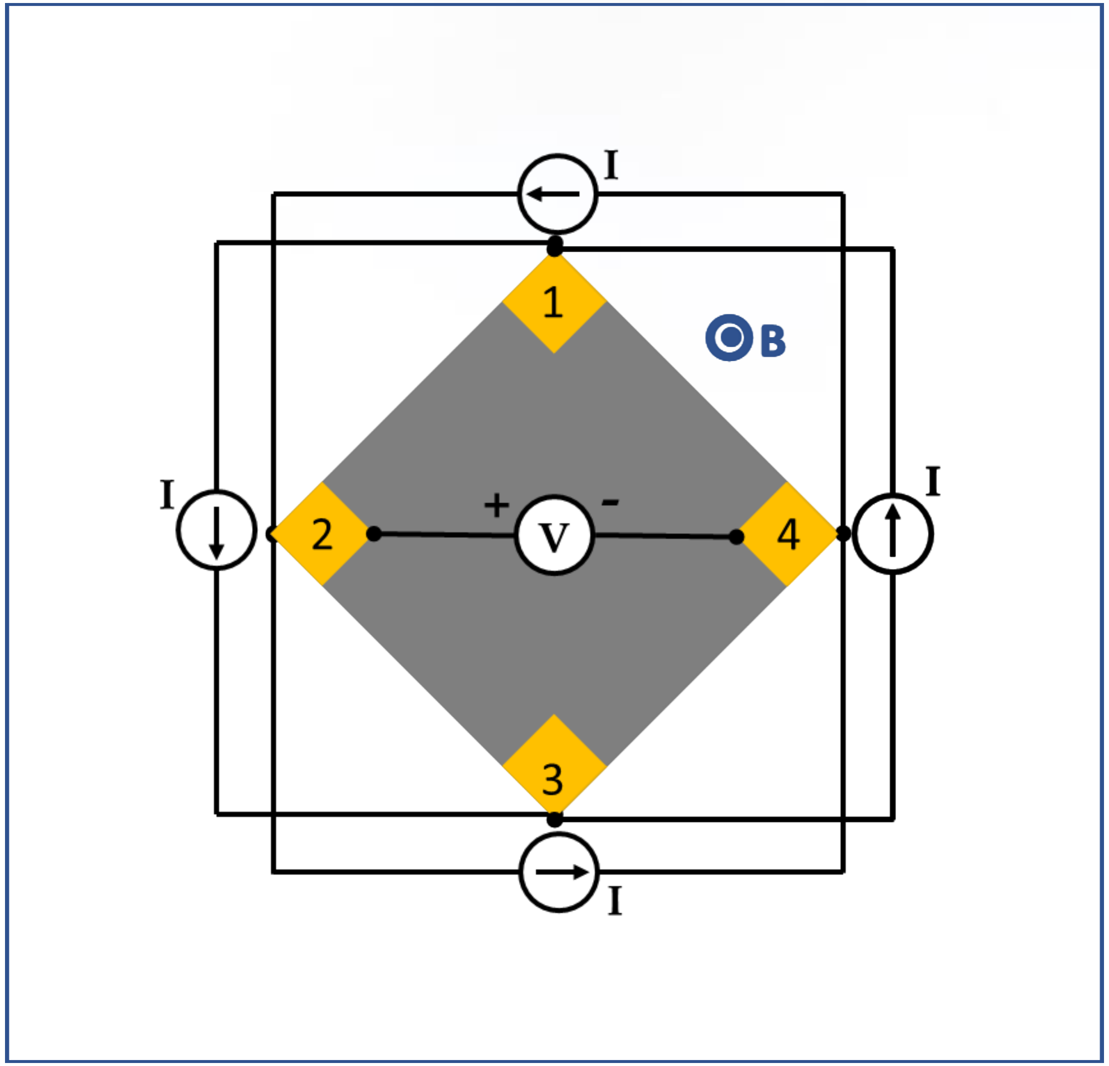


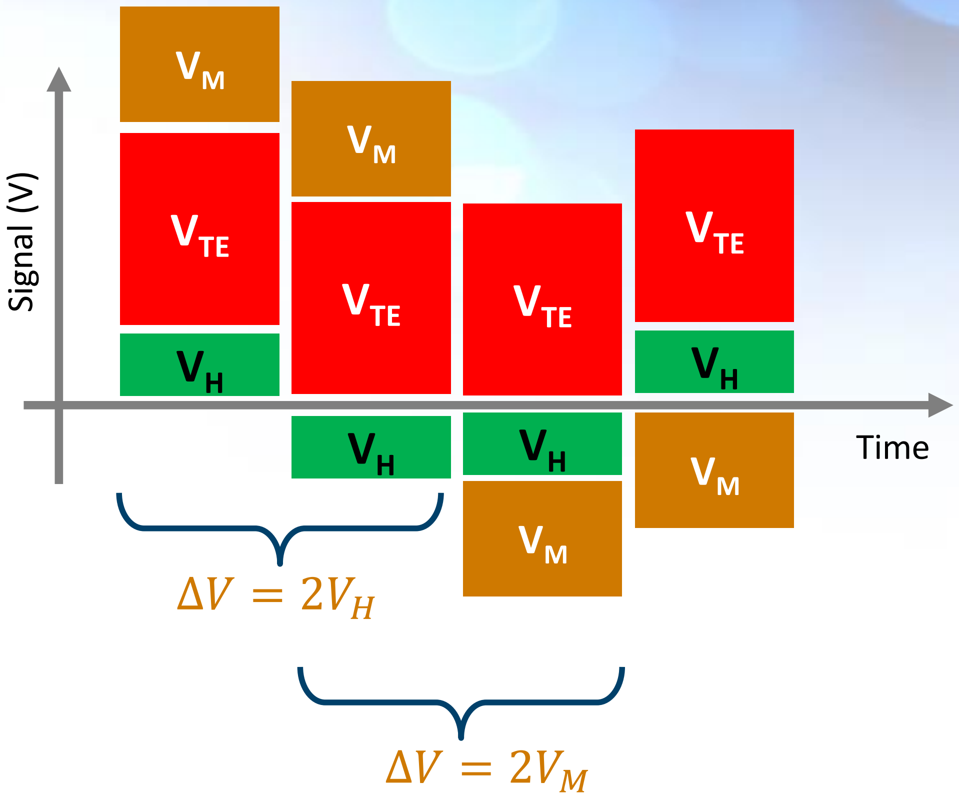
- Switch network to exchange current drive and voltage measure terminals



- For small V_H:V_M ratio, need a large dynamic range. Speed reduction

- Switch network to exchange current drive and voltage measure terminals





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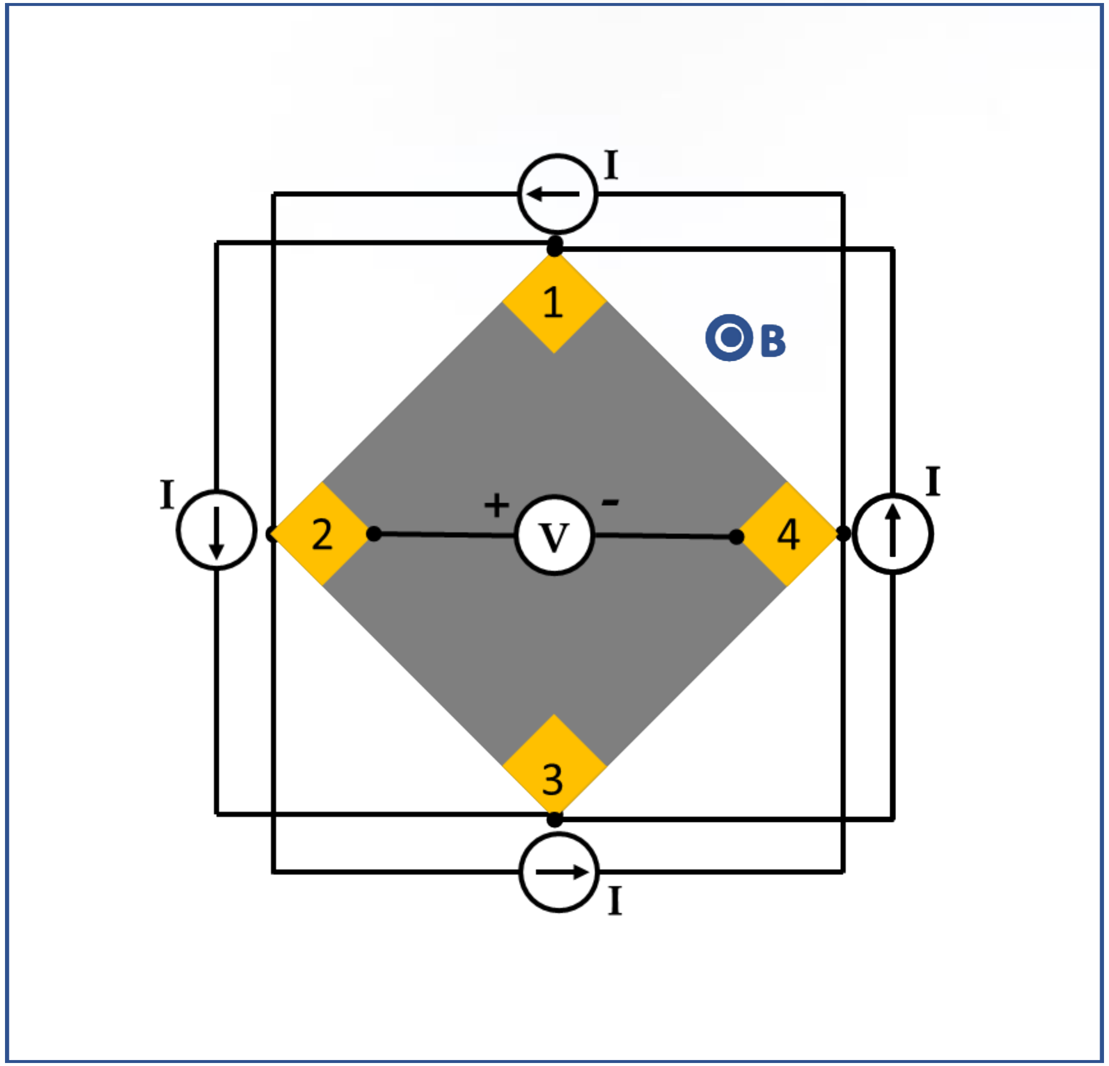

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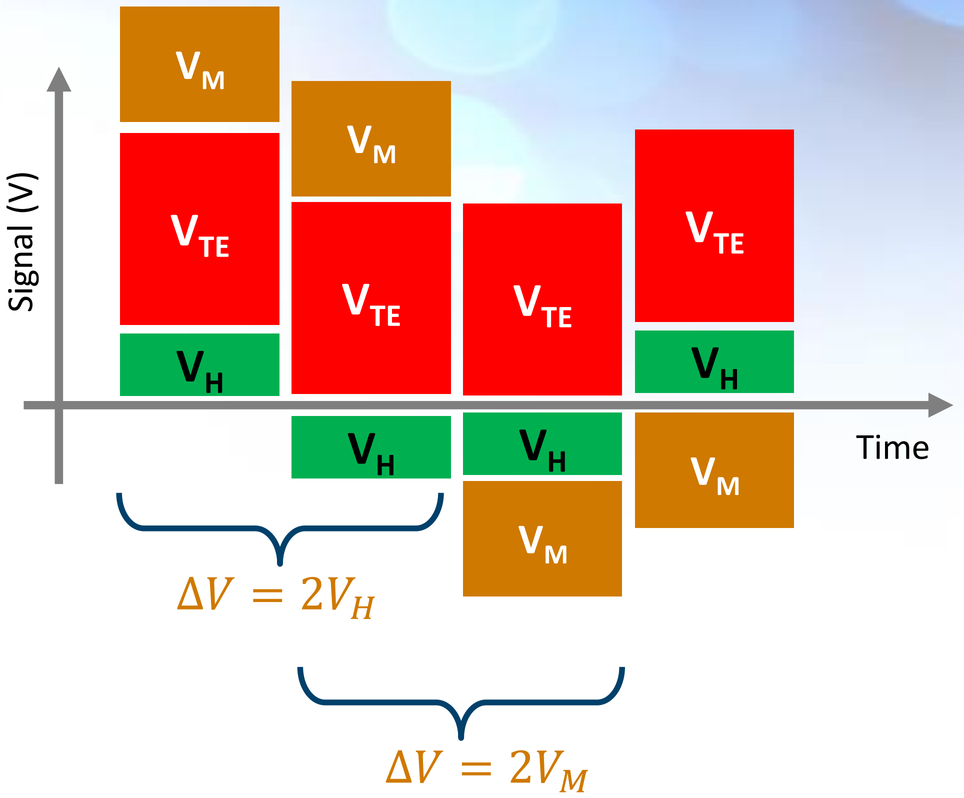

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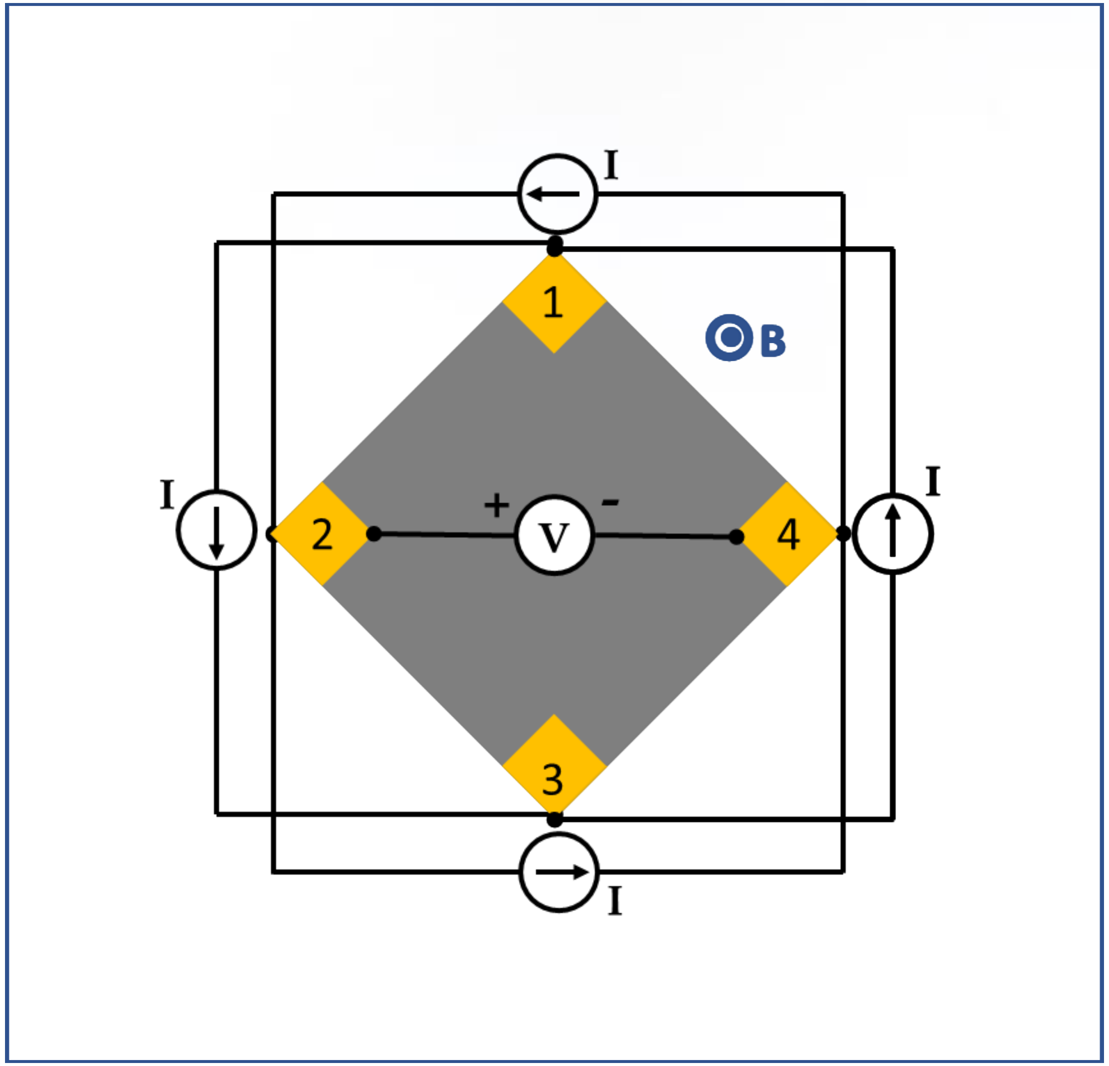

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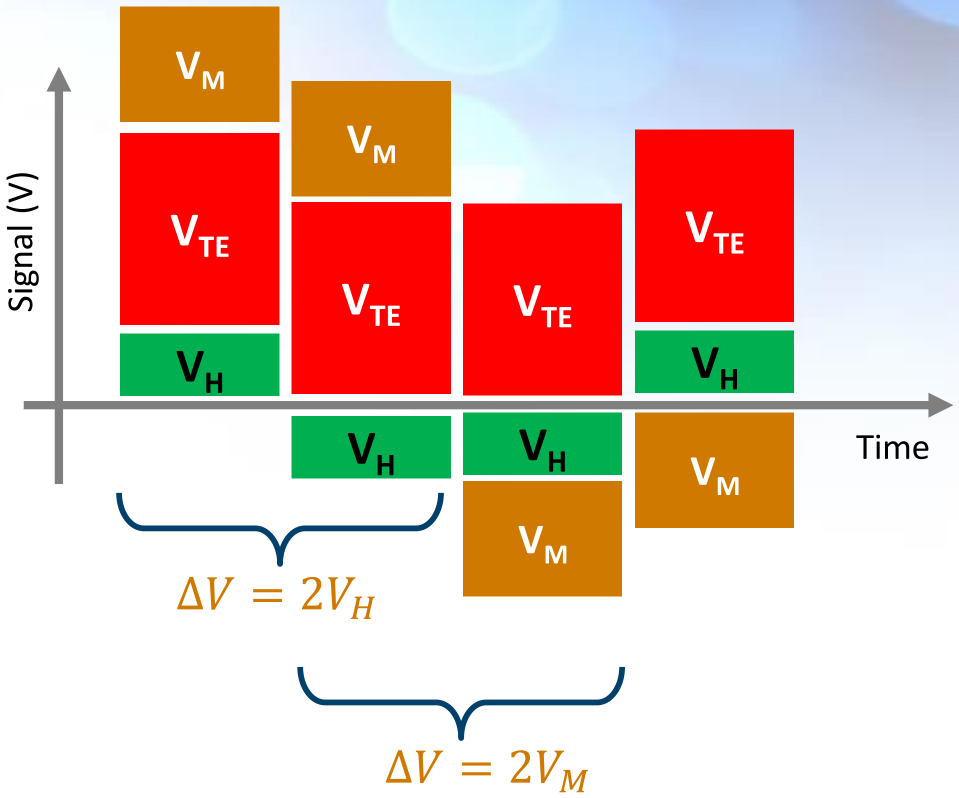

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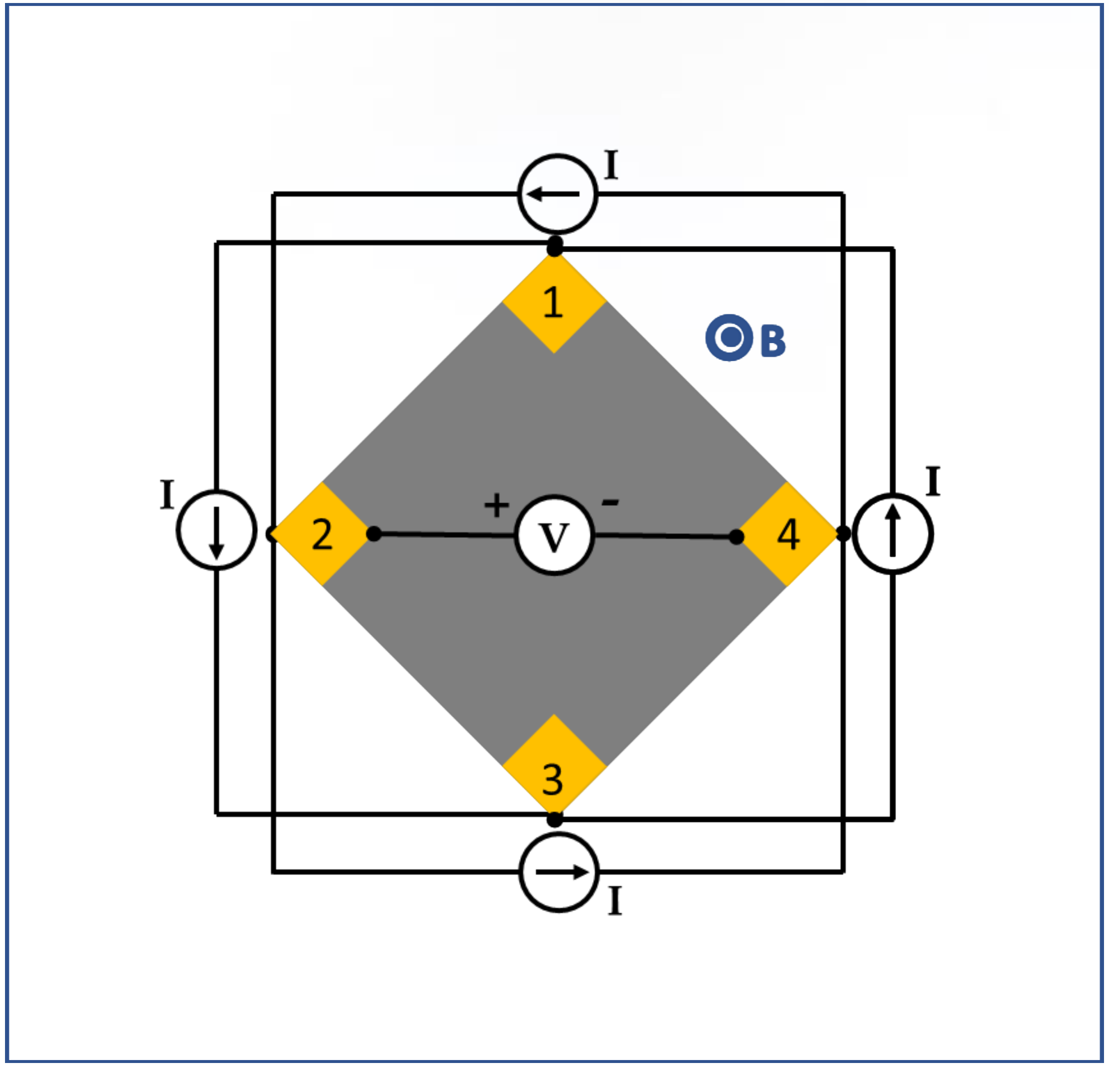

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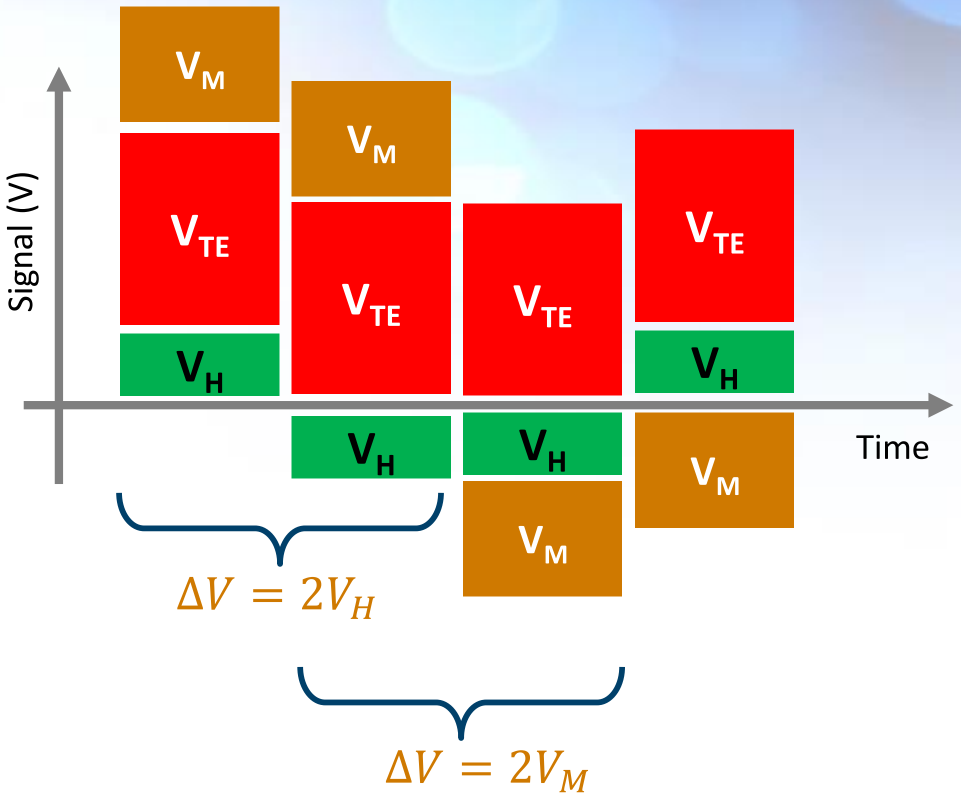

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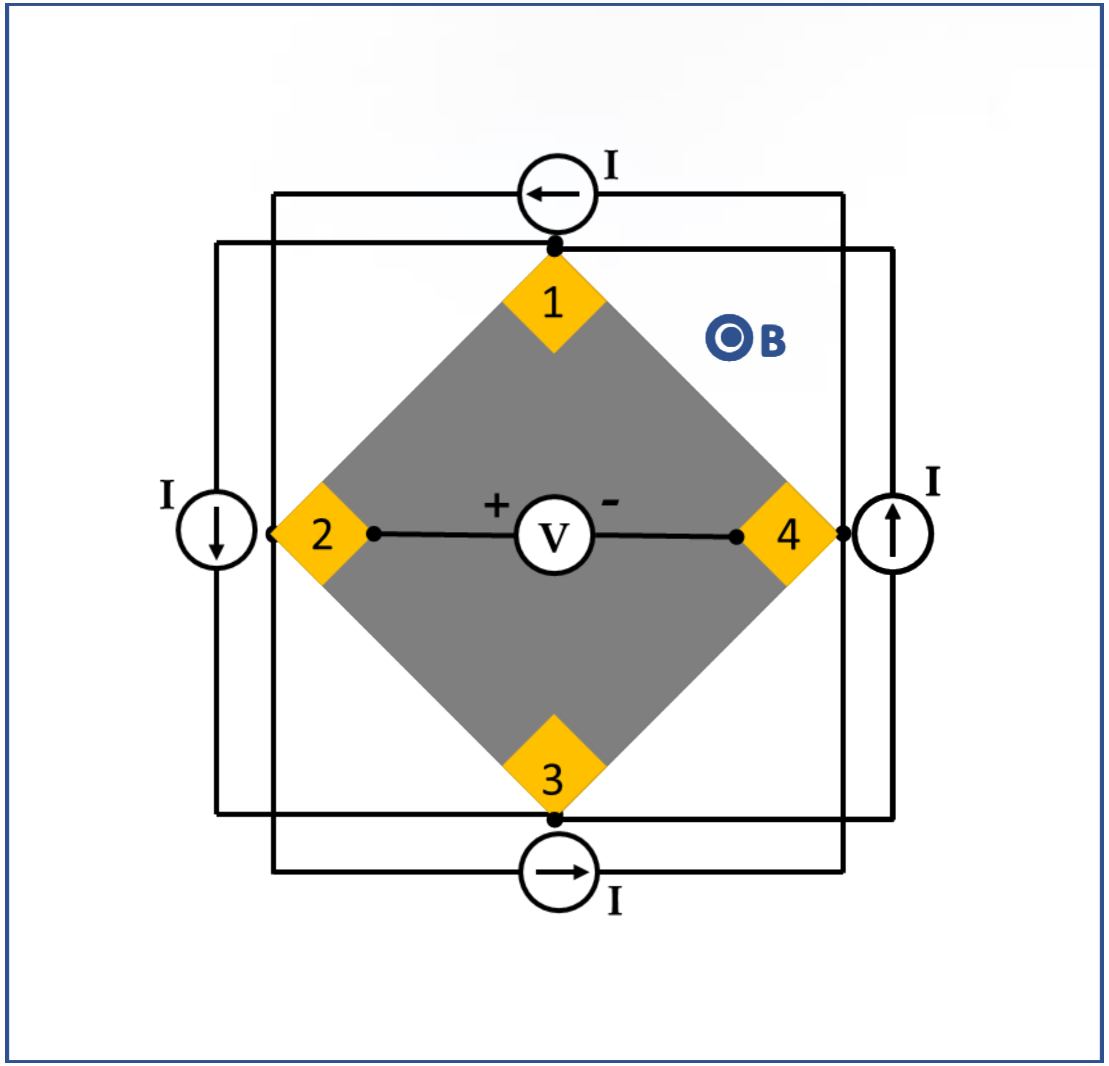

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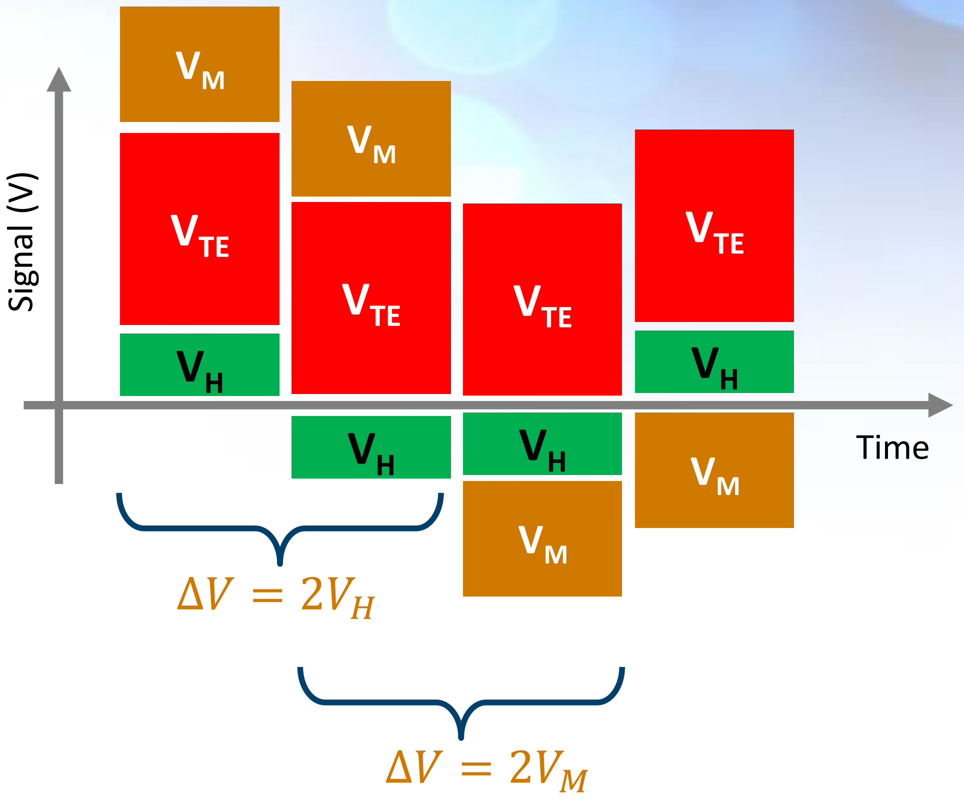


- For small V_H:V_M ratio, need a large dynamic range. Speed reduction

FastHall™ characterization

M91 FastHall controller

- Contact check, resistivity, and Hall effect measurement in one box
- Eliminates need for field reversal
  - Permanent and superconducting magnets
- Fast: 70 ms Hall measurements, 10 ms achieved
- Ideal for low mobility materials and detecting small changes in V_H

Comparisons

Low mobility samples

- 2 T DC field
- 100 samples

Sample			Conventional Hall			M91 FastHall™		
	V_M	Mobility cm²/(V s)	Hall voltage	Standard error	Time (s)	Hall voltage	Standard error	Time (s)
IGZO	4.20E-03	9.11	-0.00105	3.00E-07	114	-0.00105	5.10E-07	1.26
IGZO	2.00E-04	12.50	-9.40E-05	1.00E-06	114	-9.50E-05	2.84E-07	32.8
ZnO	1.60E-05	4.76	-8.60E-06	3.00E-08	114	-8.77E-06	6.62E-07	6.76

Ultra-low mobility samples

- 0.63 T AC or DC field
- 100 samples

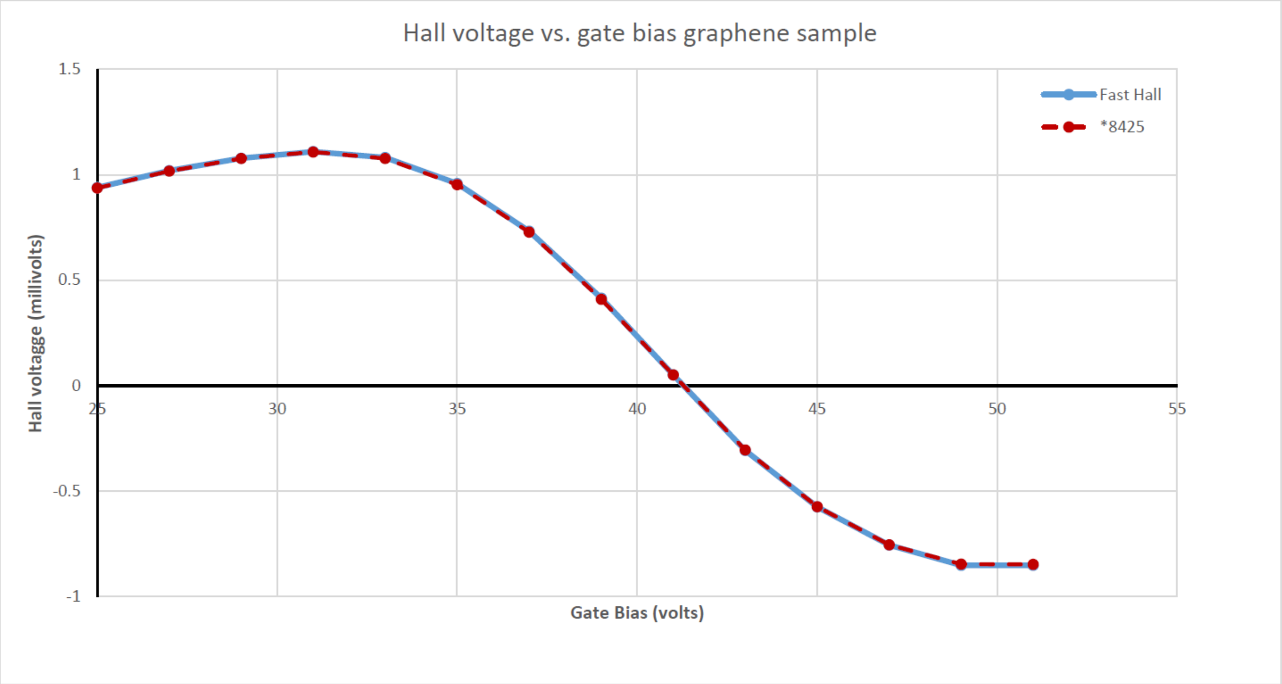
	Mobility		Resistivity	
Sample	cm²/(Vs)		ohm/sqr	
	M91	AC Hall	M91	AC Hall
Poly-Si	2.57	2.42	15.67	15.72
TiO	0.0102	0.011	1.37E+05	1.33E+05

Lake Shore CRYOTRONICS

Hall voltage measurement of graphene sample with gate bias

A gated Hall measurement introduces additional ports.

If the gate bias current is small (zero), the field reciprocity theorem still applies.



- Hall voltage vs. gate bias
  - DC field with field reversal (orange line)
  - FastHall™ method (blue line)
- Measurement time per point
  - DC Hall: 510 s
  - FastHall™: 115 s
- Thanks to Richard Kiehl of Arizona State University for providing the graphene sample