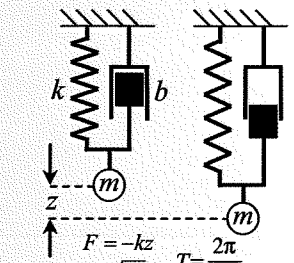
The Physics of Atomic Force Microscopy

Simple Harmonic Oscillator



$$F = -kz \qquad T = \frac{2\pi}{\omega_0}$$

$$\omega_0 = 2\pi f_0 = \sqrt{\frac{k}{m}} \qquad A \propto \exp(-t/\tau)$$

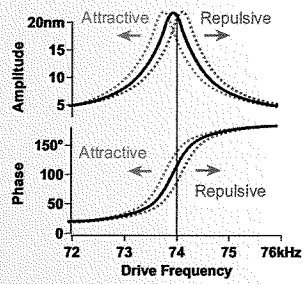
$$Q = \frac{k}{\omega_0 b} = \frac{\pi\tau}{T} = \frac{A_{max}}{A_{drive}} \cong \frac{\omega_0}{\Delta\omega}$$

$$m\frac{d^2z}{dt^2} = F_{drive} - b\frac{dz}{dt} - kz + F_{ext}$$

$$F_{drive} = kA_{drive}\cos\omega t$$

$$A = \frac{A_{drive}\omega_0^2}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + \left(\omega\omega_0/Q\right)^2}}$$

$$\tan\theta = \frac{\omega\omega_0/Q}{\omega_0^2 - \omega^2}$$



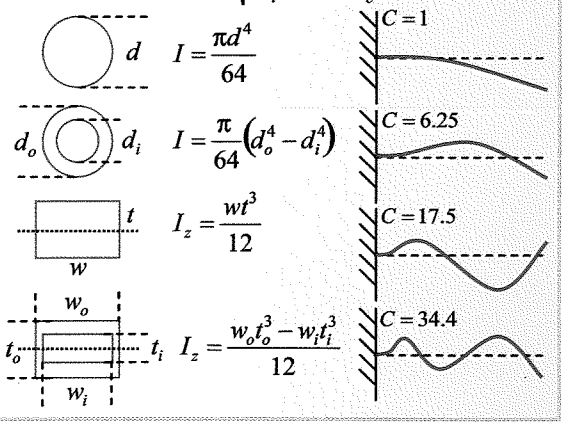
$$\delta A_{(\text{max slope})} = \frac{2QA_0}{3\sqrt{3}(k - F')}F'$$

$$\delta\omega_0 \approx -\frac{\omega_0 F'}{2k} \qquad \delta\theta_{(res)} = \frac{Q}{k}F'$$

$$\overline{P}_{tip} = \frac{1}{2}\frac{kA^2\omega}{Q}\left[\frac{QA_{drive}\sin\theta}{A} - \frac{\omega}{\omega_0}\right]$$

Cantilever Beam Theory

$$k = 3\frac{EI}{l^3} \qquad f_0 = \frac{C}{2\pi}\sqrt{\frac{k}{m_t + 0.24m_c}} \qquad m_c = \rho Al$$



Cantilever Shapes

$$h_{end}(x) = \frac{3Lx^2 - x^3}{2L^3}$$

$$h_{dist}(x) = G(\cosh\kappa x - \cos\kappa x) + H(\sinh\kappa x - \sin\kappa x)$$

Added Mass Correction

$$m_t = m_m\left(\frac{L - \Delta L}{L}\right)^3$$

Reference Spring

$$k = k_{ref}\left(\frac{\Delta z_{ref}}{\Delta z} - 1\right)$$

Shot Noise

$$\delta V_{rms} = R\sqrt{2e\eta P_0 B}$$

Johnson Noise

$$\delta V_{rms} = \sqrt{4k_B TRB}$$

Thermal Noise

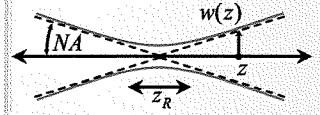
on resonance

$$\delta z_{rms} = \sqrt{\frac{4k_B TBQ}{k\omega_0}}$$

off resonance

$$\delta z_{rms} = \sqrt{\frac{4k_B TB}{Qk\omega_0}}$$

Gaussian Optics



$$w(z) = w_0\sqrt{1 + \left(\frac{\lambda z}{\pi w_0^2}\right)^2} \qquad z_R = \frac{\pi w_0^2}{\lambda} \qquad N.A. = \frac{\lambda}{\pi w_0}$$

Coherence Length

$$\Delta l = c\Delta t \cong \frac{\overline{\lambda}_0^2}{\Delta\lambda_0}$$

Thin Lens

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Energies

k_BT = 4 pNnm

ATP hydrolysis
12-21 k_BT
Hydrogen Bond
4-18 k_BT
Covalent Bond
100-200 k_BT
Protein Stability
6-20 k_BT

Forces

Covalent Bonds
2-5 nN
Unzipping DNA
20 pN (GC)/10 pN (AT)
1 mW of light reflecting off of a surface
7 pN

Distances

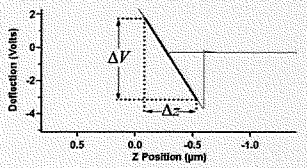
Covalent Bonds
0.1nm
Hydrogen Bond
0.25 nm
Tip Radius of Curvature
2-50 nm
DNA diameter
2nm

Stiffness

Covalent Bond
10N/m
Cantilevers
0.01 -100N/m

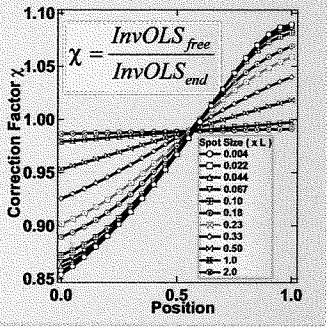
	Young's Modulus E (GPa)	Density ρ (kg/m³)	Speed of Sound (m/s)	Thermal Expansion (um/mK)	Thermal Conductivity λ (W/mK)	Heat Capacity c_p (J/gK)
Stainless	200	7760	5100	10-13	12-25	.5
Steel	210	7850	5200	11	35	.46
Ti	116	4500	5100	8.9	17	.528
Al	70	2700	5100	24	190	.88
Invar	148	8050	4300	1.3	10.15	.515
SiN	260-320	3100	~9800	3	30	.71
Si	179	2330	8800	2.6	150	.7
Cu	117	8900	3600	17	383-391	.385
W	400	19300	4500	4.4	163	.134
Granite	20-60	2500-2700	~5000	3.7-11	1.4-4.2	.21-.35
Pyrex	61	2250	5500	4.4	163	.134

Thermal Method



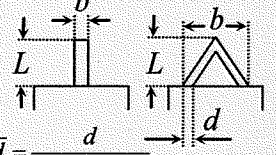
$$InvOLS_{end} = \Delta z / \Delta V (nm/volt)$$

$$k = \frac{k_B T}{\langle \delta V^2 \rangle \chi^2 InvOLS^2}$$



Sader Method

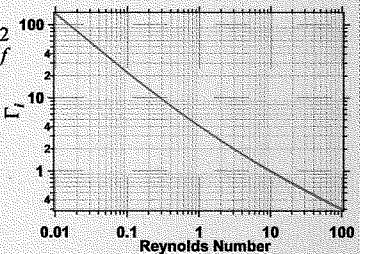
$$k_{rect} = 0.1906 b L^2 \rho_f Q_f \Gamma_i(\text{Re})\omega_f^2$$



$$\overline{d} = \frac{d}{\sqrt{1 + b^2/4L^2}}$$

$$D_0 = k_{rect}\frac{4L^3}{b} \qquad \text{Re} = \frac{\rho\omega b^2}{4\eta}$$

$$k_{tri} = \frac{D_0\overline{d}}{2L^3}\left(1 + \frac{4\overline{d}^3}{b^3}\right)^{-1}$$



$$\rho_{water} = 1x10^3 kg/m^3$$

$$\eta_{water} = 8.9x10^{-4} kg/m \cdot s$$

$$\rho_{air} = 1.18 kg/m^3$$

$$\eta_{air} = 1.86x10^{-5} kg/m \cdot s$$

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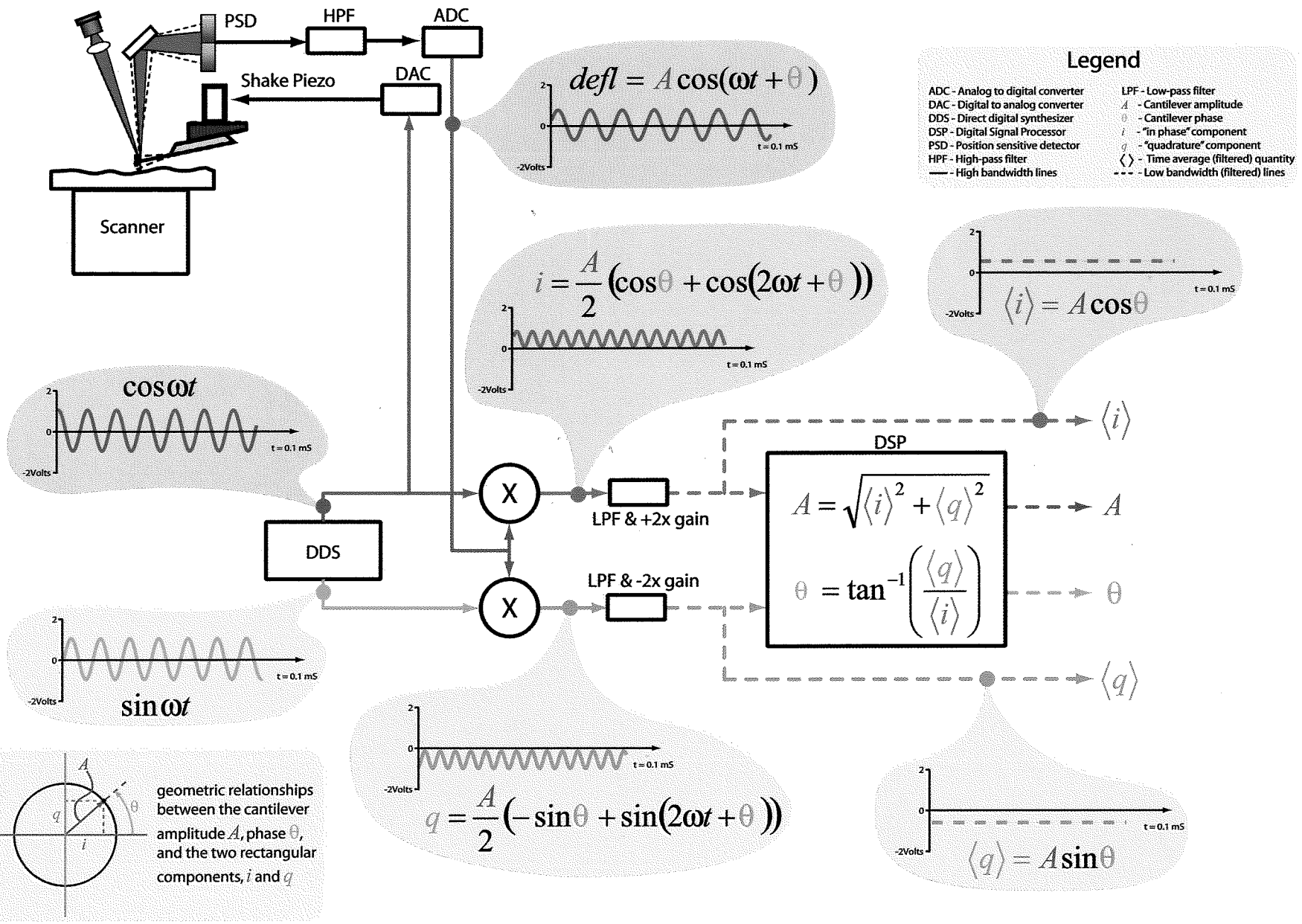
AC Digital Lock-in on the MFP-3D



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